A Study of Relationship Among Goldbach Conjecture, Twin Prime and Fibonacci Number

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Abstract

In 2015, Liu et al. proposed a relationship between RSA public key cryptosystem and Goldbach's conjecture properties. In this paper I will examine two other relationship's with Goldbach's conjecture: 1) Goldbach's conjecture and twin prime; 2) Goldbach's conjecture and Fibonacci number. I completely list all combination of twin prime in Goldbach conjecture, and propose a very simple method to recognize the prime in Fibonacci sequence. I also give a estimation formula to Goldbach's partition.

Keywords: Fibonacci number, Goldbach conjecture, twin prime

1 Introduction

The Goldbach conjecture and the twin prime issue are unsolved problems in Number Theory. It is well known that Chan [2] had a major discovery on Goldbach's conjecture by his "$1 + 2$" formal proof in 1973. Zhang [19] had a good result on the twin prime in 2014. Other articles [4–8, 10, 11, 18] also give good contributions. Liu, Chang, Wu and Ye [9] studied the relationship between RSA public key cryptosystem and Goldbach's conjecture properties. They found the RSA and Goldbach conjecture relationship, and also linked Goldbach's conjecture and twin prime. Liu et al.'s [9] listed two situations where there are twin prime numbers in Goldbach partition combinations such as Propositions 1 and 2. In addition to examining the relationship between Goldbach's conjecture and the twin prime and Fibonacci number, I will also make three major contributions:

1) Propose an estimating method which is better than Bruckman's method.

2) List all combinations of the twin prime in Goldbach's conjecture.

3) Propose a simple method to examine Fibonacci prime.

2 The Relationship Between Goldbach's Conjecture and the Twin Prime

In this section, I describe the relationship between Goldbach's conjecture and twin prime. This article is based on the work of Liu et al.'s [10] research. In Liu et al.'s article, they proposed 4 theorems, 6 propositions and 1 lemma. I continue that work and examine 6 additional situations of twin prime in Goldbach's partition. This issue is discussed in Section 2.3.

2.1 Literatures Reviews

To Goldbach's partition number, Bruckman's [1] estimated value was too large on the "number of error" range. Ye and Liu's [17] estimation is too vague, unclear and inaccurate. Based on this discussion, I give an estimating in which the number is closer to the true value. Constant [3] and Liu et al.'s [9] showed the relationship between the RSA cryptosystem and Goldbach's conjecture. Ye and Liu [17], and other articles [4, 10, 12] introduced Goldbach's conjecture and twin prime relationship. In Section 3 I will examine the relationship between Goldbach's conjecture and the Fibonacci number. The relationship between Goldbach's conjecture, twin prime, RSA and the Fibonacci number is a major topic and is shown in Figure 1. Notations are described in Table 1.

A variety of situations that may arise the twin primes in Goldbach conjecture, the all possible combination shown in Table 2.

2.2 The Goldbach Partition

Given a positive integer such as 480, there are 29 pairs to match Goldbach's rule, and 7 twin prime pairs. We say 29 is Goldbach's partition number. If randomly given an even number, it is easy to find Goldbach's partition, and we should say Goldbach's conjecture has been solved. However it is an unsolved problem today. Generally, to

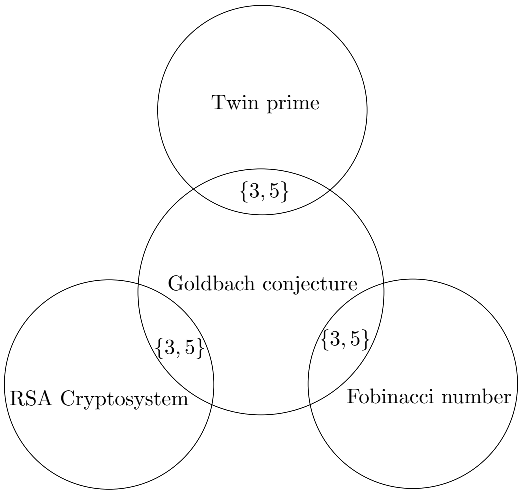

Figure 1: A relationship among Goldbach conjecture, twin prime, RSA and the Fibonacci number

Table 1: Notations

Symbols	Definition
GC	An even number for the Goldbach Conjecture (GC) number.
$GC(x)$	The number of Goldbach partition.
$GC'(x)$	The estimation number of $GC(x)$.
$GC \equiv 2\rfloor_4$	GC is congruent to two modulo four, we usually write $GC \equiv 2 \pmod 4$. But for convenience, we use $GC \equiv 2\rfloor_4$ instead here.

express GC number in the form of

$$GC = P_i + P_j \longmapsto (P_i - 2n) + (P_j + 2n), \tag{1}$$

where P_i and P_j are both primes. Let $R(n)$ be the number of representations of the Goldbach partition where $\prod_2$ is the twin prime constant [16], given $R(n) \sim 2\prod_2 \left(\prod_{P_k|n,k=2}\right) \frac{P_k-1}{P_k-2} \int_2^n \frac{dx}{(\ln x)^2}$. Ye and Liu [17] also gave the estimation formula $GC(x) = 2C\prod_{p\geq 3} \frac{(p-1)}{(p-2)} \cdot \frac{(Li(x))^2}{x} + \mathcal{O}(x \cdot e^{-c\sqrt{\ln x}})$.

In 2008, Bruckman [1] proposed a proof of the strong Goldbach conjecture, where the Goldbach function

$$\theta(2N) \equiv \sum_{k=3}^{2n-3} \delta(k)(2N-k) \tag{2}$$

is at least equal to one. Finally, the results

$$1 \leq \theta(2k+6) \leq k+1, \quad k = 0, 1, 2, \cdots. \tag{3}$$

When k approaches infinity, the error range becomes

Table 2: Twin prime appears probable in the Goldbach conjecture

item	even number				type
1	GC	$\equiv 0\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 4\rfloor_8$	$4n+2$
	$\frac{GC}{2}$	$\equiv 2\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 2\rfloor_8$	
		$\equiv 2\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 6\rfloor_8$	
2	GC	$\equiv 0\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 0\rfloor_8$	$4n$
	$\frac{GC}{2}$	$\equiv 0\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 0\rfloor_8$	
		$\equiv 0\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 4\rfloor_8$	
3	GC	$\equiv 0\rfloor_4$	$\equiv 4\rfloor_6$	$\equiv 4\rfloor_8$	$4n+2$
	$\frac{GC}{2}$	$\equiv 2\rfloor_4$	$\equiv 2\rfloor_6$	$\equiv 2\rfloor_8$	
		$\equiv 2\rfloor_4$	$\equiv 2\rfloor_6$	$\equiv 6\rfloor_8$	
4	GC	$\equiv 0\rfloor_4$	$\equiv 4\rfloor_6$	$\equiv 0\rfloor_8$	$4n$
	$\frac{GC}{2}$	$\equiv 0\rfloor_4$	$\equiv 2\rfloor_6$	$\equiv 0\rfloor_8$	
		$\equiv 0\rfloor_4$	$\equiv 2\rfloor_6$	$\equiv 4\rfloor_8$	
item	odd number				type
5	GC	$\equiv 2\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 2\rfloor_8$	$4n+1$
	$\frac{GC}{2}$	$\equiv 1\rfloor_4$	$\equiv 3\rfloor_6$	$\equiv 1\rfloor_8$	
		$\equiv 1\rfloor_4$	$\equiv 3\rfloor_6$	$\equiv 5\rfloor_8$	
6	GC	$\equiv 2\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 6\rfloor_8$	$4n+3$
	$\frac{GC}{2}$	$\equiv 3\rfloor_4$	$\equiv 3\rfloor_6$	$\equiv 3\rfloor_8$	
		$\equiv 3\rfloor_4$	$\equiv 3\rfloor_6$	$\equiv 7\rfloor_8$	
7	GC	$\equiv 2\rfloor_4$	$\equiv 4\rfloor_6$	$\equiv 2\rfloor_8$	$4n+1$
	$\frac{GC}{2}$	$\equiv 1\rfloor_4$	$\equiv 5\rfloor_6$	$\equiv 1\rfloor_8$	
		$\equiv 1\rfloor_4$	$\equiv 5\rfloor_6$	$\equiv 5\rfloor_8$	
8	GC	$\equiv 2\rfloor_4$	$\equiv 4\rfloor_6$	$\equiv 6\rfloor_8$	$4n+3$
	$\frac{GC}{2}$	$\equiv 3\rfloor_4$	$\equiv 5\rfloor_6$	$\equiv 3\rfloor_8$	
		$\equiv 3\rfloor_4$	$\equiv 5\rfloor_6$	$\equiv 7\rfloor_8$	

larger. For example:

$$\begin{aligned} \theta(32) &\leq 14,\ k = 13. \\ \theta(80) &\leq 38,\ k = 37. \\ \theta(138) &\leq 67,\ k = 66. \\ \theta(101200) &\leq 50598,\ k = 50597. \end{aligned}$$

I obtained results from a large number of experimental data. I draw the curve from the data, and then calculates the formula from the two curves (see Figures 2 and 3). I found an interesting situation which GC is congruent to zero modulo six, or congruent to non-zero modulo six. An even number GC is randomly chosen, where $GC < 6$, if $GC \equiv 0 \pmod 6$, GC' is found where $GC'(x) \simeq \frac{1.8 \cdot GC}{11.931 \cdot GC^{0.2182}}$. Otherwise, I find other $GC'(x) \simeq \frac{1.9 \cdot GC}{6.2328 \cdot GC^{0.2144}}$. The expression shown in Equation (4).

$$GC \mapsto \begin{cases} \equiv 0 \bmod 6,\ GC'(x) \simeq \frac{1.8 \cdot GC}{11.931 \cdot GC^{0.2182}}. \\ \not\equiv 0 \bmod 6,\ GC'(x) \simeq \frac{1.9 \cdot GC}{6.2328 \cdot GC^{0.2144}}. \end{cases} \tag{4}$$

I compare my estimation with Bruckman's method based on the true value of Goldbach's partition. The results indicated that my method is better than Bruckman's method, see Table 3.

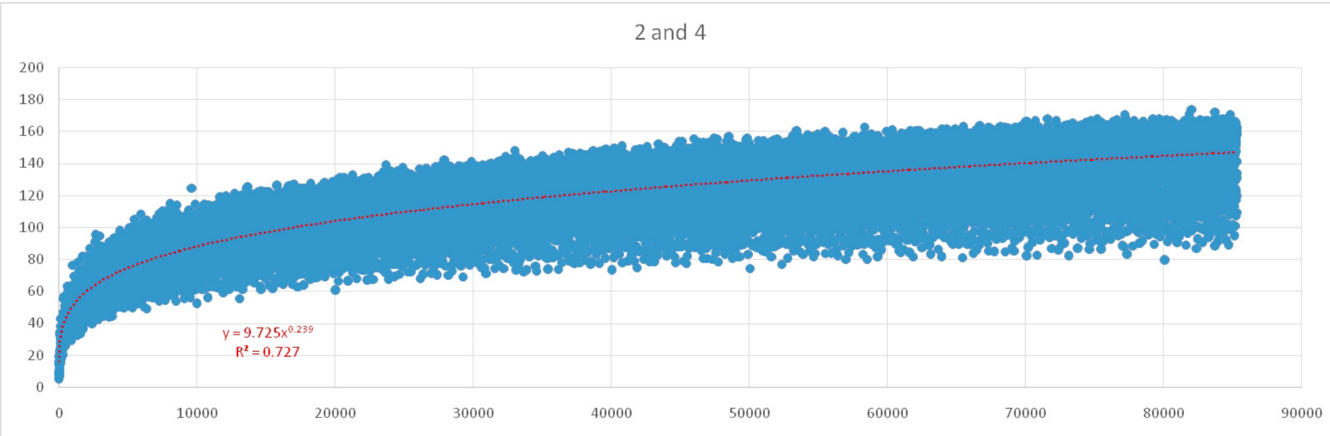

Figure 2: The curve of estimating, where $GC(x) \not\equiv 0 \pmod 6$

Table 3: Results of our method vs Bruckman's method

Item	Positive Integer	$GC(x)$	Our method	Bruckman's method	
			$GC'(x)$	k	$k+1$
1	12650	186	270	6322	6323
2	25300	314	464	12647	12648
3	50600	553	798	25297	25298
4	75900	1478	1970	37947	37948
5	101200	918	1372	50597	50598
6	126500	1140	1633	63247	63248
7	151800	2635	3396	75897	75898
8	177100	1802	2125	88547	88548
9	202400	1669	2359	101197	101198
10	227700	3688	4670	113847	113848
11	253000	2011	2808	126497	126498
12	278300	2130	3026	139147	139148
13	303600	4676	5854	151797	151798
14	318950	2059	3366	159472	159423
15	331600	2160	3470	165797	165798
16	344250	4652	6461	172122	172123
17	356900	2356	3675	178447	178448
18	369500	2321	3776	184747	184748
19	382200	6325	7015	191097	191098
20	394850	⋮	⋮	⋮	⋮
21	407500	⋮	⋮	⋮	⋮
22	420150	5264	7556	210072	210073

2.3 The Twin Prime

To help the description, I prefer to use corollary alternative propositions. These Corollaries 1 and 2 are original from Liu et al.'s [9] Propositions 1 and 2, I expand to examine 6 corollaries based on their work.

Corollary 1. *If* $P_i + P_j \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 4 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 6 \pmod 8$, *there may exist a twin prime where the* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is of the form* $(4n + 1) + (4n + 3)$.

Proof. As known from assumption, $\frac{P_i+P_j}{2}$ is an even number, we have

$$\begin{cases} \frac{P_i+P_j}{2} - 1 \text{ is an odd number.} \\ \frac{P_i+P_j}{2} + 1 \text{ is an odd number too.} \end{cases}$$

Note that $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 6 \pmod 8$, we see the $\frac{P_i+P_j}{2}$ is of the form $4n + 2$. Naturally, the $\frac{P_i+P_j}{2} - 1$ is $4n + 1$ form, and $\frac{P_i+P_j}{2} + 1$ is $4n + 3$ form. Otherwise, it is a contradiction.

Since $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8$, we know $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ is of the form $(4n + 1) + (4n + 3)$. □

Corollary 2. *If* $P_i + P_j \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 4 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is of the form* $(4n + 3) + (4n + 1)$.

Proof. As known, the $\frac{P_i+P_j}{2}$ is an even number. Since $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$. We see the $\frac{P_i+P_j}{2}$ is $4n$ form. Hence $\frac{P_i+P_j}{2} - 1$ is $4n + 3$ form. Therefore $\frac{P_i+P_j}{2} + 1$ is $4n + 1$ form.

Now, as $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$, the $\frac{P_i+P_j}{2}$ is of the form $4n$ too.

Thus, the $\frac{P_i+P_j}{2}+1$ is of the form $4n+1$. This inference is consistent with the above statement. □

Corollary 3. *If* $P_i + P_j \equiv 0 \pmod 4 \equiv 4 \pmod 6 \equiv 4 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 2 \pmod 6 \equiv 2 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 2 \pmod 6 \equiv 6 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is of the form* $(4n + 3) + (4n + 1)$.

Proof. As known from assumption, the $\frac{P_i+P_j}{2} \equiv 2 \pmod 4$ is an even number. Unsurprisingly, the $\frac{P_i+P_j}{2} - 1$ is $4n+1$ form. Hence, the $\frac{P_i+P_j}{2}+1$ would be $4n+3$ form. Otherwise, it is a contradiction. This inference is consistent with the above statement. □

Corollary 4. *If* $P_i + P_j \equiv 0 \pmod 4 \equiv 4 \pmod 6 \equiv 0 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 2 \pmod 6 \equiv 0 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 2 \pmod 6 \equiv 4 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is of the form* $(4n + 3) + (4n + 1)$.

Proof. As known, the $\frac{P_i+P_j}{2}$ is an even number. Since $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 2 \pmod 6 \equiv 0 \pmod 8$, the $\frac{P_i+P_j}{2}$ is an even number and a $4n$ form. Obviously, the $\frac{P_i+P_j}{2} - 1$ is $4n + 3$ form, whereas the $\frac{P_i+P_j}{2} + 1$ is of the form $4n + 1$. Otherwise, it is a contradiction. This inference is consistent with the above statement. □

Corollary 5. *If* $P_i + P_j \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 1 \pmod 4 \equiv 3 \pmod 6 \equiv 1 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 1 \pmod 4 \equiv 3 \pmod 6 \equiv 5 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is of the form* $(4n + 3) + (4n + 1)$.

Proof. As known, the $\frac{P_i+P_j}{2} \equiv 1 \pmod 4$, the $\frac{P_i+P_j}{2}$ is $4n+1$ form. Since $4n+1$ and $4n+3$ are located on either side of the center point $4n + 2$. Thus, the $(\frac{P_i+P_j}{2} + 2)$ is of the form $4n + 3$. If not, it is a contradiction. □

Corollary 6. *If* $P_i + P_j \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 6 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 3 \pmod 4 \equiv 3 \pmod 6 \equiv 3 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 3 \pmod 4 \equiv 3 \pmod 6 \equiv 7 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is of the form* $(4n + 3) + (4n + 1)$.

Proof. This proof is same with Corollary 5. I omit the proof here. □

Corollary 7. *If* $P_i + P_j \equiv 2 \pmod 4 \equiv 4 \pmod 6 \equiv 2 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 1 \pmod 4 \equiv 5 \pmod 6 \equiv 1 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 1 \pmod 4 \equiv 5 \pmod 6 \equiv 5 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is of the form* $(4n + 3) + (4n + 1)$.

Proof. This proof is same with Corollary 5. I also omit the proof here. □

Corollary 8. *If* $P_i + P_j \equiv 2 \pmod 4 \equiv 4 \pmod 6 \equiv 6 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 3 \pmod 4 \equiv 5 \pmod 6 \equiv 3 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 3 \pmod 4 \equiv 5 \pmod 6 \equiv 7 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is of the form* $(4n + 3) + (4n + 1)$.

Proof. This proof is same with Corollary 5. I omit the proof here too. □

Exception:
There are 4 exceptions of even number between $[2, 1000]$ to the rule in Table 2.

$$402 \mapsto \begin{cases} 402 \equiv 2 \bmod 4 \equiv 0 \bmod 6 \equiv 2 \bmod 8, \\ 201 \equiv 1 \bmod 4 \equiv 3 \bmod 6 \equiv 1 \bmod 8. \end{cases} \quad (5)$$

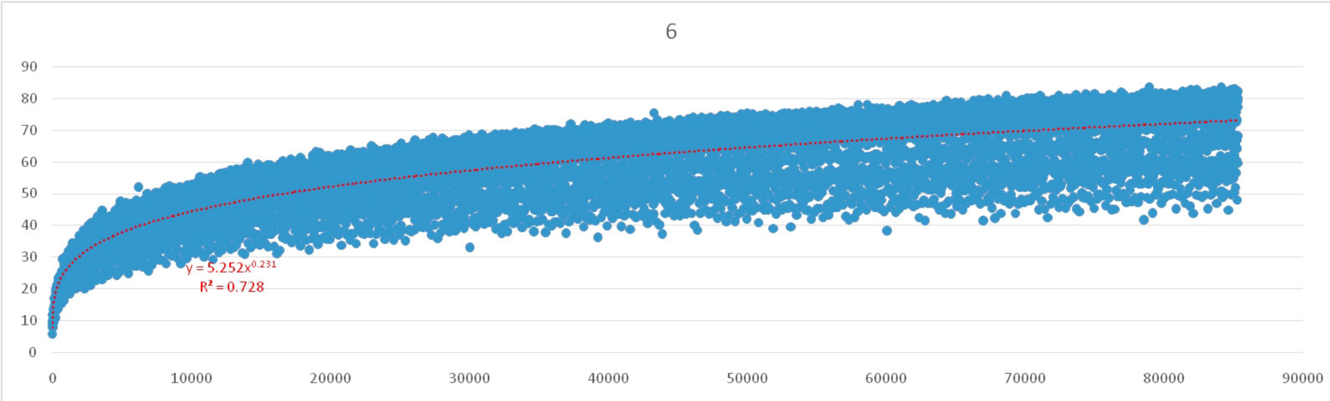

Figure 3: The curve of estimating, where $GC(x) \equiv 0 \pmod 6$

According from Table 2, the 402 matches item 5, however, there is no one twin prime in 17 prime pairs of Goldbach partition.

$$516 \mapsto \begin{cases} 516 \equiv 0 \bmod 4 \equiv 0 \bmod 6 \equiv 4 \bmod 8, \\ 258 \equiv 2 \bmod 4 \equiv 0 \bmod 6 \equiv 2 \bmod 8. \end{cases} \quad (6)$$

There are 23 prime pairs in Goldbach partition, but no one matches in the rule of item 1.

$$786 \mapsto \begin{cases} 786 \equiv 2 \bmod 4 \equiv 0 \bmod 6 \equiv 2 \bmod 8, \\ 393 \equiv 1 \bmod 4 \equiv 3 \bmod 6 \equiv 1 \bmod 8. \end{cases} \quad (7)$$

There are 30 prime pairs in Goldbach partition, but no one matches in the rule of item 5.

$$906 \mapsto \begin{cases} 906 \equiv 2 \bmod 4 \equiv 0 \bmod 6 \equiv 2 \bmod 8, \\ 453 \equiv 1 \bmod 4 \equiv 3 \bmod 6 \equiv 5 \bmod 8. \end{cases} \quad (8)$$

There are 34 prime pairs in Goldbach partition, but no one matches in the rule of item 5.

3 The Relationship of the Goldbach's Conjecture and the Fibonacci Number

This section will introduce about Fibonacci number [14, 15] and it's relationship with Goldbach's conjecture. Each positive number is the sum of the previous two integers, namely

$$F_n = F_{n-1} + F_{n-2}. \quad (9)$$

By Equation (9), we know the Fibonacci sequence as $\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \cdots, \infty\}$. Wall [13] had good result in his article "Fibonacci Series Modulo M", a table was created in the appendix listing values for the function $k(n)$. This function is defined as the period of the Fibonacci numbers mod n before any repeats occur. For instance, $k(7) = 16$ since

$$F_n \bmod 7 = \{0, 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1\}, \quad (10)$$

where F_n is the n-th Fibonacci number. Hence, the values in the sequence above are cyclic after 16 terms. On the other hand, the author notes another interesting property. The Fibonacci sequence has 'even-odd-odd' or 'odd-odd-even' rotation rules. The result shown in Table 4.

For n-th Fibonacci number, where $n \geq 1$, the F_n becomes an odd number if and only if $n \equiv 1 \pmod 3$ or $n \equiv 2 \pmod 3$, say

$$n \begin{cases} \equiv 0 \pmod 3\text{, this is an even number.} \\ \equiv 1 \pmod 3\text{, this is an odd number.} \\ \equiv 2 \pmod 3\text{, this is an odd number.} \end{cases}$$

There is one example of the Fibonacci number matching the Goldbach's rule where the

$$F_6 = F_5 + F_4 \mapsto 3 + 5 = 8. \quad (11)$$

The Equation (11) is only one special case of Goldbach's conjecture in Fibonacci sequence nowaday. Since $F_{n\equiv 0 \pmod 3}$ has never been an prime that is an even number, we can say the $F_{n\equiv 1 \pmod 3}$ or $F_{n\equiv 2 \pmod 3}$ probable is a prime. There is an article by Wall [13] about Fibonacci prime in [14], but is a little different then what is discussed in this article.

Open Problems:

Can we find the second example of Goldbach's conjecture in Fibonacci sequence? In Fibonacci prime, I find an interesting phenomenon in my research.

1. If $n \equiv 3 \pmod 4$ and $F_n \equiv 1 \pmod 4$ where $n > 5$, the F_n probably be a prime, where
$$\begin{cases} F_{n\equiv 3 (\bmod 4)} \\ F_n \equiv 1 \pmod 4 \end{cases} \quad (12)$$
2. If $n \equiv 1 \pmod 4$ and $F_n \equiv 1 \pmod 4$ where $n > 5$, the F_n probably be also a prime, namely
$$\begin{cases} F_{n\equiv 1 (\bmod 4)} \\ F_n \equiv 1 \pmod 4 \end{cases} \quad (13)$$

We get following relationship as:

Goldbach's conjecture $\supseteq$ $(odd + odd = even) \subset$ Fibonacci sequence.

Table 4: The special case of Fibonacci number matches the Goldbach's conjecture

				prime	prime	prime		prime				prime		prime	
n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
F_n	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377
		odd	odd	even	odd	odd	even	odd	odd	even	odd	odd	even	odd	odd
$F_n \equiv X$ mod7	0	1	1	2	3	5	1	6	0	6	6	5	4	2	6

			prime						prime						prime
n	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
F_n	610	987	1597	2584	4181	6765	10946	17711	28657	46368	75025	121393	196418	317811	514229
	even	odd	odd	even	odd	odd	even	odd	odd	even	odd	odd	even	odd	odd
$F_n \equiv X$ mod7	1	0	1	1	2	3	5	1	6	0	6	6	5	4	2

n	30	31	32	33	34	35	36	37	38	39
F_n	832040	1346269	2178309	3524578	5702887	9227465	14930352	24157817	39088169	63245986
	even	odd	odd	even	odd	odd	even	odd	odd	even
$F_n \equiv X$ mod7	6	1	0	1	1	2	3	5	1	6

				prime				prime		
n	40	41	42	43	44	45	...	81839		
F_n	102334155	165580141	267914296	433494437	701408733	1134903170	...	17103 digits		
	odd	odd	even	odd	odd	even				
$F_n \equiv X$ mod7	0	6	6	5	4	2		1		

4 Conclusions

I use Goldbach's conjecture as the center of interest. I then discusses the relationship among Goldbach's conjecture, twin prime, RSA cryptosystem and Fibonacci number and then makes three observations about the relationship:

1) The characteristics of twin prime in Goldbach's conjecture are analyzed, and then notes all situations of combination.

2) An estimate of Goldbach's partition is proposed where the result is more accurate than Bruckman's estimation.

3) Finally, I explore the relationship between Goldbach's conjecture and Fibonacci number. I mention a new discussion about searching the Fibonacci prime in its sequence.

As we can see, the authors is still working on these unsolved problems.

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Biography

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