

Stability Analysis of a Worm Propagation Model with Quarantine and Vaccination

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Abstract

Internet worms pose a serious threat to the Internet security. In order to effectively defend against Internet worms, this paper proposes a novel epidemic e - $SEIQV$ model with quarantine and vaccination. Using this e - $SEIQV$ model, we obtain the basic reproduction number for determining whether the worm dies out completely. The global stability of the worm-free equilibrium and the local stability of endemic equilibrium are proved, and determined by the basic reproduction number. Besides the impact of different parameters of this model is studied. Simulation results show that the number of susceptible and infected hosts are consistent with the theoretical analysis. The model provides a theoretical foundation for controlling and forecasting Internet worms.

Keywords: Endemic equilibrium, internet worm, network security, stability analysis, vaccination

1 Introduction

Internet worms are malicious codes which can replicate themselves and propagate via Internet. With the ever increasing number of Internet applications and the emergence of new technologies, Internet worms have become a great threat to our work and daily life, and caused tremendous economic losses. Especially, the advent of the Internet of things (IoT) would make the threat increasingly serious. How to combat Internet worms effectively is an urgent issue confronted with defenders. Therefore, it is necessary to comprehend the long-term behavior of worms and to propose effective strategies to defend against In-

ternet worms. The similarity between the spread of biological viruses and that of Internet worms encourages researchers to adopt appropriately modified epidemic models to describe the propagation of worms across the Internet.

Based on the similarity between a malicious worm and a biological virus, some epidemic models representing worm propagation were presented to depict the propagation of worms, e.g., SIR model [17], $SIRS$ model [10, 16], SIQ model [27], $SEIR$ model [11], $SEIRS$ model [15, 19], $SEIQV$ model [20], $SEIQRS$ model [28, 9], which assume that infected hosts in which the worm resides are in an exposed state and can not infect other hosts. Actually, an infected host which is in latency can infect other hosts by means of some methods, e.g., vulnerability seeking. All the previous models do not take this passive infectivity into consideration. Recently, Yang et al. [22, 23, 24, 25] proposed some models, by taking into account the fact that a host immediately possesses infectivity once it is infected. These models, however, all make an assumption that exposed hosts and infected hosts have the same infectivity. This is not consistent with the reality. Although an exposed host also sends scanning packets to find susceptible hosts with certain vulnerabilities, the number of scanning packets sent by an exposed host is less than that of an infected one. Usually, the infection rate of exposed hosts is less than that of infected ones. Therefore, they should have different infection rates.

Due to the frequent occurrence of worms over the Internet in the last decade, users usually install some antivirus softwares or firewalls to protect their hosts. Once a user feels that the performance of his host is degraded or there exists some useless data in disks (e.g., Witty can

do it), he will clean worms by antivirus softwares. In order to protect his important files, the user spontaneously clean worms even if he is not sure the existence of worms in his host. Additionally, an infected host often represents more obvious characteristics than an exposed host, the user could take some more effective measurements, e.g., patching. Therefore, the cured rate of exposed hosts would be lower than that of infected ones. The feature should be considered when modeling Internet worms. Recently, more attention has been paid to the combination of worm propagation model and countermeasures to study the prevalence of worms, e.g., quarantine [11, 20] and vaccination [4, 5, 14].

In this paper, we propose a new worm attack model, referred to as *e-SEIQV* (Susceptible - Exposed - Infectious - Quarantined - Vaccinated) model, which incorporates the features mentioned above. Using the basic reproduction number, we derive the global stabilities of a worm-free equilibrium and a unique endemic equilibrium by a Lyapunov function and a geometric approach. Based on these results and further analysis, some effective methods for controlling worms are recommended.

The rest of this paper is organized as follows. Section 2 formulates the new model and obtain its basic reproduction number. Section 3 proves the global stabilities of the worm-free equilibrium and the endemic equilibrium. Section 4 covers the numerical analysis and the simulations. Section 5 summarizes the paper with some future directions.

2 Model Formulation

The total host population N is partitioned into five groups and any host can potentially be in any of these groups at any time tick t : the susceptible, exposed (latent), infectious, quarantined, vaccinated, with sizes denoted by S, E, I, Q, V , respectively. The total number of population N at time t is given by $N(t) = S(t) + E(t) + I(t) + Q(t) + V(t)$. The dynamical transfer of hosts is depicted in the following figure.

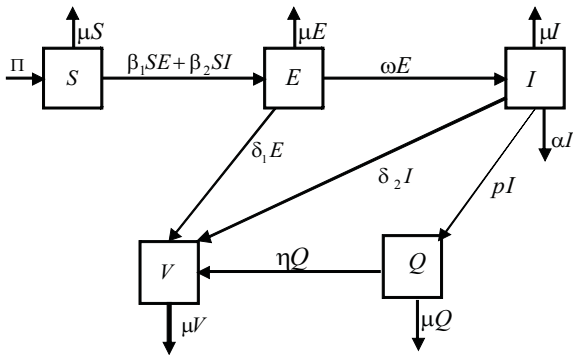


Figure 1: Schematic diagram for the flow of Internet worms

Figure 1 shows the five states and state transition in

e-SEIQV. Based on the compartment model presented in Figure 1, the *e-SEIQV* model having infectious force in the exposed, infected period is described by the following system of differential equations:

$$\begin{cases} S'(t) = \Pi - \beta_1SE - \beta_2SI - \mu S, \\ E'(t) = \beta_1SE + \beta_2SI - (\mu + \delta_1 + \omega)E, \\ I'(t) = \omega E - (\mu + \alpha + \delta_2 + p)I, \\ Q'(t) = pI - (\eta + \mu)Q, \\ V'(t) = \delta_1E + \delta_2I + \eta Q - \mu V, \end{cases} \quad (1)$$

where Π is a constant recruitment of susceptible hosts. β_1, β_2 are the rates of the efficient contact in the latent, infected period, respectively. The positive parameter μ is the rate of natural death, α are non-negative constant and denote the rate of worm-caused death. $\delta_1, \delta_2, \omega$ are the transfer rates between the exposed and the vaccinated, between the infectious and the vaccinated, between the exposed and the infectious, respectively. The parameter p denotes the quarantined rate. The parameter η denotes the transfer rate between the quarantined and the vaccinated.

Summing the equations of the system (1), we obtain

$$N(t)' = \Pi - \mu N - \alpha I. \quad (2)$$

Therefore, the total population N may vary with time t . In the absence of disease, the total population size $N(t)$ converges to the the equilibrium Π/μ . It follows from Equation (2) that $\liminf_{t \rightarrow \infty} N(t) \leq \Pi/\mu$. We thus study our system (1) in the following feasible region:

$$\Omega = \{(S, E, I, Q, V) \in \mathbb{R}_+^5 : S + E + I + Q + V \leq \Pi/\mu\},$$

which is a positively invariant set of Model (1). We next consider the dynamic behavior of Model (1) on Ω .

Firstly, we obtain the basic reproduction number of Model (1) by the method of next generation matrix [1]. It is easy to see that Model (1) always has a worm-free equilibrium, $P_0 = (\Pi/\mu, 0, 0, 0, 0)$.

Let $x = (E, I, Q, V, S)^T$, then Model (1) can be written as

$$\frac{dx}{dt} = \mathcal{F}(x) - \mathcal{V}(x),$$

where

$$\mathcal{F}(x) = \begin{pmatrix} \beta_1SE + \beta_2SI \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\mathcal{V}(x) = \begin{pmatrix} (\mu + \delta_1 + \omega)E \\ (\mu + \alpha + \delta_2 + p)I - \omega E \\ (\eta + \mu)Q - pI \\ \mu V - \delta_1E - \delta_2I - \eta Q \\ \beta_1SE + \beta_2SI + \mu S - \Pi \end{pmatrix}.$$

Differentiating $\mathcal{F}(x)$ and $\mathcal{V}(x)$ with respect to E, I, Q, V, S and evaluating at the worm-free equilibrium $P_0 = (\Pi/\mu, 0, 0, 0, 0)$, respectively, we have

$$D\mathcal{F}(P_0) = \begin{pmatrix} F_{2 \times 2} & 0_{2 \times 3} \\ 0_{3 \times 2} & 0_{3 \times 3} \end{pmatrix},$$

$$D\mathcal{V}(P_0) = \begin{pmatrix} Y_{2 \times 2} & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ Y'_{3 \times 2} & \eta + \mu & 0 & 0 \\ & -\eta & \mu & 0 \\ & 0 & 0 & \mu \end{pmatrix},$$

where

$$F_{2 \times 2} = \begin{pmatrix} \frac{\beta_1 \Pi}{\mu} & \frac{\beta_2 \Pi}{\mu} \\ 0 & 0 \end{pmatrix}, Y'_{3 \times 2} = \begin{pmatrix} 0 & -p \\ -\delta_1 & -\delta_2 \\ \frac{\beta_1 \Pi}{\mu} & \frac{\beta_2 \Pi}{\mu} \end{pmatrix},$$

and

$$Y_{2 \times 2} = \begin{pmatrix} \mu + \delta_1 + \omega & 0 \\ -\omega & \mu + \alpha + \delta_2 + p \end{pmatrix}.$$

Thus, the spectral radius of the next generation matrix \mathcal{FV}^{-1} can be found as,

$$\rho(\mathcal{FV}^{-1}) = \frac{\Pi(\beta_1(\mu + \alpha + \delta_2 + p) + \beta_2\omega)}{\mu(\mu + \alpha + \delta_2 + p)(\mu + \delta_1 + \omega)}.$$

According to Theorem 2 in [1], the basic reproduction number of Model (1) is

$$R_0 = \frac{\Pi(\beta_1(\mu + \alpha + \delta_2 + p) + \beta_2\omega)}{\mu(\mu + \alpha + \delta_2 + p)(\mu + \delta_1 + \omega)}. \tag{3}$$

For the concision of notation, let $m = \mu + \alpha + \delta_2 + p$ and $n = \mu + \delta_1 + \omega$. Thus $R_0 = \frac{\Pi(\beta_1 m + \beta_2 \omega)}{\mu mn}$.

The endemic equilibrium $P^*(S^*, E^*, I^*, Q^*, V^*)$ of Model (1) is determined by equations

$$\begin{cases} \Pi - \beta_1 SE - \beta_2 SI - \mu S = 0, \\ \beta_1 SE + \beta_2 SI - nE = 0, \\ \omega E - mI = 0, \\ pI - (\eta + \mu)Q = 0, \\ \delta_1 E + \delta_2 I + \eta Q - \mu V = 0, \end{cases} \tag{4}$$

and

$$\Pi - \mu N - \alpha I = 0. \tag{5}$$

By some simple computation, we obtain

$$\begin{cases} S = \frac{\Pi \omega \alpha}{(\beta_1 m + \beta_2 \omega)(\Pi - \mu N) + \alpha \omega \mu}, \\ E = \frac{m(\Pi - \mu N)}{\omega \alpha}, \\ I = \frac{\Pi - \mu N}{\alpha}, \\ Q = \frac{p(\Pi - \mu N)}{\alpha(\eta + \mu)}, \\ V = \frac{[\delta_1 m(\eta + \mu) + \omega \delta_2(\eta + \mu) + \omega \eta p](\Pi - \mu N)}{\omega(\eta + \mu)\mu \alpha}. \end{cases}$$

Substituting Equation (6) into the second equation of the system (1), N satisfies $F(N)(\Pi - \mu N) = 0$, where, $F(N) = [\mu mnN - \Pi(mn - \omega\alpha)](\beta_1 m + \beta_2 \omega) - \mu mn \omega \alpha$.

For $R_0 > 1$, $F(0) = -\Pi(mn - \omega\alpha)(\beta_1 m + \beta_2 \omega) - \mu mn \omega \alpha < 0$ and $F(\Pi/\mu) = \mu mn \omega \alpha (R_0 - 1)$, thus $F(N)$ is monotone increasing and $F(\Pi/\mu) > 0$. Within the interval $(0, \Pi/\mu)$, $F(N)$ has only a positive root. That is, Model (1) has an unique endemic equilibrium $P^*(S^*, E^*, I^*, Q^*, V^*)$, where S^*, E^*, I^*, Q^*, V^* are determined by Equation (6).

3 Analysis of Model

3.1 Global Stability of P_0

It is easily obtained that the model has a worm-free equilibrium given by $P_0 = (\Pi/\mu, 0, 0, 0, 0)$.

Lemma 1. *When $R_0 < 1$, the worm-free equilibrium P_0 is locally asymptotically stable in Ω . When $R_0 > 1$, the worm-free equilibrium P_0 is an unstable saddle point.*

Proof. The Jacobian matrices of Model (1) at P_0 is

$$J(P_0) = \begin{pmatrix} -\mu & -\frac{\beta_1 \Pi}{\mu} & -\frac{\beta_2 \Pi}{\mu} & 0 & 0 \\ 0 & \frac{\beta_1 \Pi}{\mu} - n & \frac{\beta_2 \Pi}{\mu} & 0 & 0 \\ 0 & \omega & -m & 0 & 0 \\ 0 & 0 & p & -(\eta + \mu) & 0 \\ 0 & \delta_1 & \delta_2 & \eta & -\mu \end{pmatrix}$$

It is easily obtained that $J(P_0)$ has three negative eigenvalues $\lambda_1 = \lambda_2 = -\mu$, and $\lambda_3 = -(\eta + \mu)$, the other eigenvalues of $J(0)$ are determined by the following equation:

$$\lambda^2 + (m + n - \beta_1 \Pi/\mu)\lambda + mn - (m\beta_1 + \omega\beta_2)\Pi/\mu = 0. \tag{7}$$

When $R_0 < 1$, $mn > (m\beta_1 + \omega\beta_2)\Pi/\mu$.

For $mn > (m\beta_1 + \omega\beta_2)\Pi/\mu$, we can obtain $m + n > m + \beta_1 \Pi/\mu + \Pi\beta_2 \omega/(\mu m)$, thus $m + n - \beta_1 \Pi/\mu > m + \beta_2 \Pi/\mu > 0$, which means the Equation (7) has two negative roots. Therefore, the worm-free equilibrium P_0 is locally asymptotically stable.

When $R_0 > 1$, $mn - (m\beta_1 + \omega\beta_2)\Pi/\mu < 0$, which means the Equation (7) has a positive root and a negative root. Therefore, the worm-free equilibrium P_0 is unstable saddle point. \square

Lemma 2. *When $R_0 \leq 1$, the worm-free equilibrium P_0 is globally asymptotically stable in Ω . When $R_0 > 1$, all solutions starting in Ω and sufficiently close to P_0 move away from P_0 .*

Proof. Consider the Lyapunov function

$$L = \frac{\beta_1 m + \beta_2 \omega}{mn} E + \frac{\beta_2}{m} I.$$

Its derivative along the solutions to Model (1) is

$$\begin{aligned} L' &= \frac{\beta_1 m + \beta_2 \omega}{mn} (\beta_1 SE + \beta_2 SI - nE) + \frac{\beta_2}{m} (\omega E - mI) \\ &= \frac{\beta_1 m + \beta_2 \omega}{mn} (\beta_1 SE + \beta_2 SI) - (\beta_1 E + \beta_2 I) \\ &= (\beta_1 E + \beta_2 I) \left(\frac{\beta_1 m + \beta_2 \omega}{mn} S - 1 \right) \\ &\leq (\beta_1 E + \beta_2 I) \left(\frac{\Pi(\beta_1 m + \beta_2 \omega)}{mn\mu} - 1 \right) \\ &= (\beta_1 E + \beta_2 I)(R_0 - 1) \\ &\leq 0. \end{aligned}$$

Furthermore, $L' = 0$ if and only if $E = I = 0$ or $R_0 = 1$. Thus, the largest compact invariant set in $\{(S, E, I, Q, V) | L' = 0\}$ is the singleton $\{P_0\}$. When $R_0 \leq 1$, the global stability of P_0 follows from LaSalle's invariance principle [6]. LaSalle's invariance principle [6] implies that P_0 is globally asymptotically stable in Ω . When $R_0 > 1$, it follows from the fact $L' > 0$ if $E > 0$ and $I > 0$. This completes the proof. \square

3.2 Global Stability of P^*

Lemma 3. *When $R_0 > 1$, the endemic equilibrium P^* is locally asymptotically stable in Ω .*

Proof. Replacing S with $N - E - I - Q - V$ in Model (1), we obtain

$$\begin{cases} E'(t) = (\beta_1 E + \beta_2 I)(N - E - I - Q - V) - nE, \\ I'(t) = \omega E - mI, \\ Q'(t) = pI - (\eta + \mu)Q, \\ V'(t) = \delta_1 E + \delta_2 I + \eta Q - \mu V, \\ N'(t) = \Pi - \mu N - \alpha I = 0. \end{cases} \quad (8)$$

The Jacobian matrices of Model (8) at $P^* = (E^*, I^*, Q^*, V^*, N^*)$ is

$$J(P^*) = \begin{pmatrix} a\beta_1 - b - n & a\beta_2 - b & -b & -b & b \\ \omega & -m & 0 & 0 & 0 \\ 0 & p & -c & 0 & 0 \\ \delta_1 & \delta_2 & \eta & -\mu & 0 \\ 0 & -\alpha & 0 & 0 & -\mu \end{pmatrix}$$

where, $a = \frac{mn}{\beta_1 m + \beta_2 \omega}$, $b = \beta_1 E + \beta_2 I$, and $c = \eta + \mu$. Its characteristic equation is $\det(\lambda I - J(P^*)) = 0$, where I is the unit matrix. Therefore,

$$\begin{aligned} \det(\lambda I - J(P^*)) &= (\lambda_1 + \mu)(\lambda_2 + (\eta + \mu))(\lambda^3 + A\lambda^2 + B\lambda + C) = 0, \end{aligned}$$

where,

$$\begin{aligned} A &= b + m + \mu + n - \frac{\beta_1 mn}{\beta_1 m + \beta_2 \omega} \\ &= b + \mu + m + \frac{\beta_2 n \omega}{\beta_1 m + \beta_2 \omega} > 0, \\ B &= b(\delta_1 + m + \mu + \omega) + m\mu + mn \\ &\quad + n\mu - \left(\frac{\beta_1 m^2 n + \beta_1 m n \mu + \beta_2 m n \omega}{\beta_1 m + \beta_2 \omega} \right) \\ &= b(\delta_1 + m + \mu + \omega) + m\mu + \frac{\beta_2 \omega n \mu}{\beta_1 m + \beta_2 \omega} > 0, \end{aligned}$$

$$\begin{aligned} C &= b(m\delta_1 + \alpha\omega + m\mu + \delta_2\omega + \omega\mu + p\omega) \\ &\quad + mn\mu - \frac{\beta_1 m^2 n \mu + \beta_2 m n \mu \omega}{\beta_1 m + \beta_2 \omega} \\ &= b(m\delta_1 + \alpha\omega + m\mu + \delta_2\omega + \omega\mu + p\omega) > 0. \end{aligned}$$

By a direct calculation, we obtain that $AB - C > 0$. According to the theorem of Routh-Hurwitz, the endemic equilibrium P^* is locally asymptotically stable. \square

For Model (8), we consider global stability of the endemic equilibrium P^* when $\alpha = 0$. Since $\liminf_{t \rightarrow \infty} N(t) \leq \Pi/\mu$, Model (9) is a four-dimensional asymptotically autonomous differential system with limit system

$$\begin{cases} E'(t) = (\beta_1 E + \beta_2 I) \left(\frac{\Pi}{\mu} - E - I - Q - V \right) - nE, \\ I'(t) = \omega E - mI, \\ Q'(t) = pI - (\eta + \mu)Q, \\ V'(t) = \delta_1 E + \delta_2 I + \eta Q - \mu V. \end{cases} \quad (9)$$

Next, we apply the geometrical approach [6] to investigate the global stability of the endemic equilibrium P^* in the region Ω .

Theorem 1. [6] *Consider the following systems:*

$x' = f(x)$, $x \in \Omega$. *If the following conditions are satisfied:*

- 1) *The system (*) exists a compact absorbing set $K \subset \Omega$ and has a unique equilibrium P^* in Ω ;*
- 2) *P^* is locally asymptotically stable;*
- 3) *The system (*) satisfies a Poincaré-Bendixson criterion;*
- 4) *A periodic orbit of the system (*) is asymptotically orbitally stable, then the only equilibrium P^* is the globally asymptotically stable in Ω .*

Lemma 4. *If $R_0 > 1$, the unique positive equilibrium P^* of Model (9) is globally asymptotically stable in Ω .*

Proof. We only need to prove that all assumptions of Theorem 1 hold.

If $R_0 > 1$, then the worm-free equilibrium is unstable according to Lemma 1. Moreover, the behavior of the local dynamics near the region P_0 described in Lemma 1 implies that Model (9) is uniformly persistent in the region Ω . That is, there exists a constant $c > 0$, such that any solution $(E(t), I(t), Q(t), V(t))$ of Model (9) with initial value $(E(0), I(0), Q(0), V(0))$ in Ω satisfies

$$\min\{\liminf_{t \rightarrow \infty} E(t), \liminf_{t \rightarrow \infty} I(t), \liminf_{t \rightarrow \infty} Q(t), \liminf_{t \rightarrow \infty} V(t)\} \geq c.$$

This can be proved by applying a uniform persistent result in [3] and by the use of a similar argument as in the proof in [7]. The uniform persistence of system (9) in the bounded set Ω is equivalent to the existence of a compact $K \subset \Omega$ that is absorbing for system (9). In Section 3, during the process of obtaining the endemic equilibrium

P^* , we can know that P^* is the unique equilibrium in the interval $(0, \Pi/\mu)$. Assumption (1) holds.

According to Lemma 3, we know that the endemic equilibrium P^* is locally asymptotically stable in the region Ω . Assumption (2) holds.

The Jacobian matrix of Model (9) is denoted by

$$J(P^*) = \begin{pmatrix} \beta_1 S - b - n & \beta_2 S - b & -b & -b \\ \omega & -m & 0 & 0 \\ 0 & p & -c & 0 \\ \delta_1 & \delta_2 & \eta & -\mu \end{pmatrix}. \quad (10)$$

Choosing the matrix H as $H = \text{diag}(1, -1, 1, 1)$, it is easy to prove that HJH has non-positive off-diagonal elements, thus we can obtain that system (9) is competitive. This verifies the assumption (3).

The second compound matrix $J^{[2]}(P^*)$ of $J(P^*)$ can be calculated as follows:

$$J^{[2]}(P^*) = \begin{pmatrix} A1 & 0 & 0 & b & b & 0 \\ p & A2 & 0 & A & 0 & b \\ \delta_2 & \eta & A3 & 0 & A & -b \\ 0 & \omega & 0 & A4 & 0 & 0 \\ -\delta_1 & 0 & \omega & \eta & A5 & 0 \\ 0 & -\delta_1 & 0 & -\delta_2 & p & A6 \end{pmatrix} \quad (11)$$

where, $A = \beta_2 S - b$, $A1 = -(b + n + m - \beta_1 S)$, $A2 = -(b + n + \eta + \mu - \beta_1 S)$, $A3 = -(b + n + \mu - \beta_1 S)$, $A4 = -(m + \eta + \mu)$, $A5 = -(m + \mu)$, $A6 = -(\eta + 2\mu)$.

The second compound system of Model (9) in a periodic solution can be represented by the following differential equations:

$$\begin{cases} X'(t) = A_1 X + bL + bM, \\ Y'(t) = pX + A_2 Y - (b - \beta_2 S)L + bU, \\ Z'(t) = \delta_2 X + \eta Y + A_3 Z - (b - \beta_2 S)M - bU, \\ L'(t) = \omega Y - (m + \eta + \mu)L, \\ M'(t) = -\delta_1 X + \omega Z + \eta L - (m + \mu)M, \\ U'(t) = -\delta_1 Y - \delta_2 L + pM - (\eta + 2\mu)U. \end{cases} \quad (12)$$

In order to prove that the system (12) is asymptotically stable, we consider the following Lyapunov function:

$$V(X, Y, Z, L, M, U; E, I, Q, V) = \sup\{|X| + |Y| + |Z|, \frac{E}{I}(|L| + |M| + |U|)\}.$$

According to the uniform persistence, we obtain that the orbit of $P(t) = (E(t), I(t), Q(t), V(t))$ remains a positive distance from the boundary of Ω , thus, we know that there exists a constant c satisfying

$$V(X, Y, Z, L, M, U; E, I, Q, V) \geq c \sup\{|X|, |Y|, |Z|, |L|, |M|, |U|\},$$

for all $(X, Y, Z, L, M, U) \in \mathbb{R}^6$ and $(E, I, Q, V) \in P(t)$.

For the differential equations in Equation (12), we obtain the following differential inequalities by direct calculations:

$$D_+(|X| + |Y| + |Z|)$$

$$\leq -(2\mu + \delta_1 + \omega)(|X| + |Y| + |Z|) + \frac{E}{I}(\beta_1 S + \beta_2 S \frac{I}{E})(|L| + |M| + |U|),$$

$$D_+(|X| + |Y| + |Z|)$$

$$\leq -(2\mu + \delta_1 + \omega)(|X| + |Y| + |Z|) + \frac{E}{I}(\beta_1 S + \beta_2 S \frac{I}{E})(|L| + |M| + |U|),$$

Then,

$$D_+ \frac{E}{I}(|L| + |M| + |U|) \leq \omega \frac{E}{I}(|X| + |Y| + |Z|) + (\frac{E'}{E} - \frac{I'}{I} - (2\mu + \alpha + \delta_2 + p)) \frac{E}{I}(|L| + |M| + |U|).$$

From the pervious formula, we can obtain

$$D_+ |V(t)| \leq \max\{g_1(t), g_2(t)\} V(t),$$

where,

$$g_1(t) = -(2\mu + \delta_1 + \omega) + (\beta_1 S + \beta_2 S \frac{I}{E}),$$

$$g_2(t) = \omega \frac{E}{I} + \frac{E'}{E} - \frac{I'}{I} - (2\mu + \alpha + \delta_2 + p).$$

From Model (1), we can obtain

$$\frac{E'}{E} = \beta_1 S + \beta_2 S \frac{I}{E} - (\mu + \delta_1 + \omega),$$

$$\frac{I'}{I} = \omega \frac{E}{I} - (\mu + \alpha + \delta_2 + p).$$

Therefore,

$$g_1(t) = \frac{E'}{E} - \mu, g_2(t) = \frac{E'}{E} - \mu.$$

Then,

$$\int_0^\zeta \sup\{g_1(t), g_2(t)\} dt \leq \ln E(t)|_0^\zeta - \mu\zeta = -\mu\zeta < 0,$$

which implies that $(X(t), Y(t), Z(t), L(t), M(t), U(t)) \rightarrow 0$, as $t \rightarrow \infty$. Thus, the second compound system (12) is asymptotically stable. This verifies the assumption (4).

We verify all the assumptions of Theorem 1. Therefore, P^* is globally asymptotically stable in Ω . \square

4 Numerical Simulations

In this experiment, we choose the Code Red as basic behavior of a worm. The Code Red infected 360,000 hosts on July 19th 2001 [12], thus 360,000 hosts are selected as the population size. According to the real conditions of the Code Red worm, the worm's average scan rate is $s = 358$ per minute. Code Red worm's infection rate can then be computed as $\beta_2 = s/2^{32} = 8.34 \times 10^{-8}$, $\beta_1 = 8 \times 10^{-8}$. At the beginning, the number of susceptible, exposed, infected, quarantined and vaccinated hosts are $S(0) = 359,985$, $E(0) = 5$, $I(0) = 10$, $Q(0) = 0$ and $V(0) = 0$, respectively. The quarantined rate of infected hosts is $p = 0.2$ per minute, i.e., on average an infected host can propagate for about 5 minutes before it is alarmed and quarantined.

Other parameters in these simulations are given as follows: $\Pi = 2,160$, $\mu = 0.006$, $\gamma = 0.03$, $\theta = 0.03$, $\alpha = 0.005$, $\delta_1 = 0.02$, $\delta_2 = 0.04$, $\omega = 0.005$, $\eta = 0.005$, where $R_0 = 0.9873 < 1$. The worm will gradually disappear according to Theory 2. Figure 2 illustrates the number of susceptible and infected hosts when R_0 is 0.9873. From Figure 2, we can clearly see that the tendency of the worm propagation is depressive, which is consistent with Lemma 2. Finally, the whole population, in the long term, is in a vaccinated state. In order to effectively defend against such worms, we must adopt some feasible methods to decrease the infection rate [18, 21] or increase the following parameters (e.g., the transfer rates between the exposed and the recovered, between the exposed and the infectious) to guarantee the basic reproduction number $R_0 < 1$.

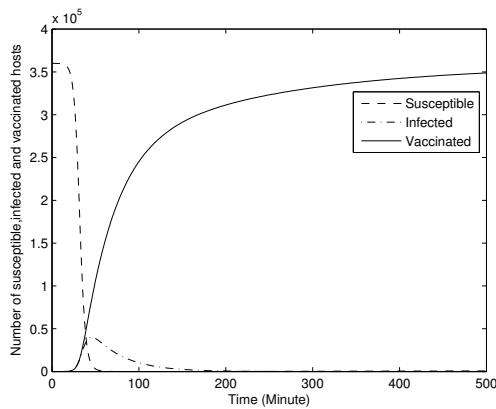


Figure 2: Globally asymptotically stable worm-free equilibrium

In the second experiment, the number of susceptible, exposed, infected, quarantined and vaccinated hosts are $S(0) = 359,985$, $E(0) = 5$, $I(0) = 10$, $Q(0) = 0$ and $V(0) = 0$, respectively. When $\delta_2 = 0.01$, $p = 0.02$, $\omega = 0.08$, we can obtain $R_0 = 6.9397 > 1$. For $\delta_2 = 0.01$, $p = 0.02$, $\omega = 0.04$, $R_0 = 10.5718 > 1$. For $\delta_2 = 0.01$, $p = 0.02$, $\omega = 0.02$, $R_0 = 11.2284 > 1$. Other parameters do not vary. We can see the results in Figure 3. As

can be seen from Figure 3, the number of susceptible and infected hosts eventually become positive values between 0 and Π/μ . $S(t)$, $I(t)$ all approach their steady state, and the worm persists. This is fully consistent with the conclusions of Lemma 4.

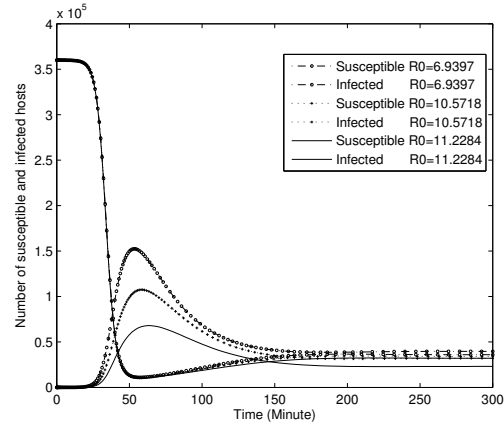


Figure 3: Globally asymptotically stable endemic equilibrium

With other parameters remaining the same, the quarantined rate p is set to different value at each time in order to state that the number of infected hosts is affected by every different value of the quarantined rate. Figure 4 shows the effects of changing the quarantined rate (which vary between 0.1 and 0.9) on worm propagations. As expected, a larger quarantined rate results in diminishing the worm propagation speed, and lowering the total number of infected hosts. Quarantined rate p relies mainly on the accuracy and detection speed of intrusion detection algorithms. Some methods have been proposed to reach the goal, e.g., a pulse quarantine strategy [26], an orchestration approach [2].

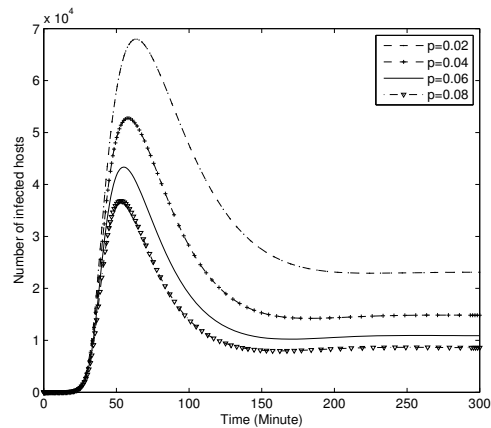


Figure 4: Effect of the quarantined rate p on the number of infected hosts

5 Conclusion

This paper presented a mathematical model to describe the dynamical behavior of an e -SEIQV epidemic model with quarantine and vaccination for Internet worms. Firstly, by the method of next generation matrix, we give the basic reproduction number to determine whether the worm extinguishes. Secondly, the global asymptotic stabilities of our model have been proved by using the Lyapunov function and a geometric approach. When the basic reproduction number is less than or equal to one, the proposed model has only a worm-free equilibrium being globally stable, which implies the worm dies out eventually; when the basic reproduction number is larger than one, our model has a unique endemic equilibrium being globally stable, which implies that the worm persists in the whole host population and tends to a steady state. Finally, some numerical examples are given to verify our conclusions. Our future work will expand this model to characterize more features of Internet worms, e.g., taking delay or impulse into consideration.

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